Points-based rules respecting a pairwise-change-symmetric ordering

AMS Special Session on Voting Theory

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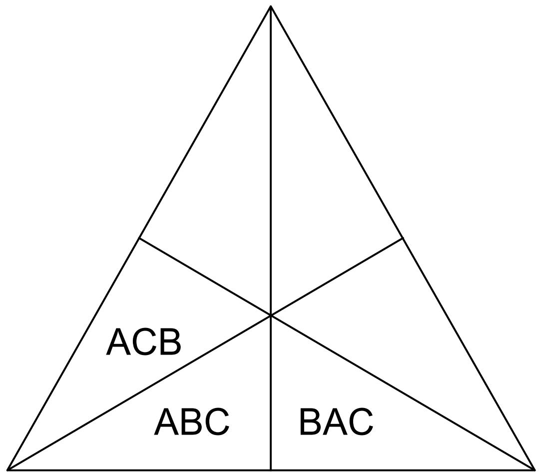
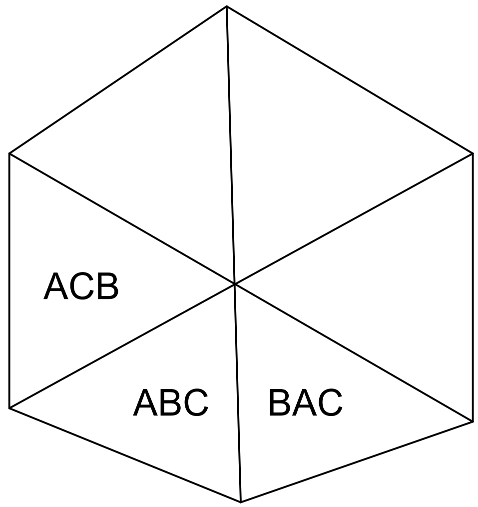
A simple idea

Our usual representations of profiles favor the winner/first choice candidate, so that the transition from ABC to BAC is implicitly bigger than from ABC to ACB.

But it is also natural to consider that *any* pair switch which does not affect any other pair should be weighted equally, as in Dodgson's rule.





A simple idea

- The common way to order outcomes is lexicographic (which biases winners).
  - For example, a voter of type ABCD would prefer outcome ADCB to BACD.
- However, simple pairwise exchanges provide another (partial) order.
  - The same voter of type ABCD would prefer BACD to ADCB.

A simple idea

This view is implicit in recent work:

- In Terao’s (2006) proof of Arrow’s Theorem, a simple swap corresponds to moving through a minimum codimension face in the braid arrangement.
- Zwicker’s work with spatial/physical models places the unanimity profiles at the corners of a regular hexagon.

A simple idea

- We suggest a family of social welfare functions similar to points-based procedures, but assigning points to each *ranking*, not each candidate.
- These points will be symmetric with respect to this partial order, with rules like “x-y-x” or “x-y-z-y-x”.
- It’s easier to do than to explain!

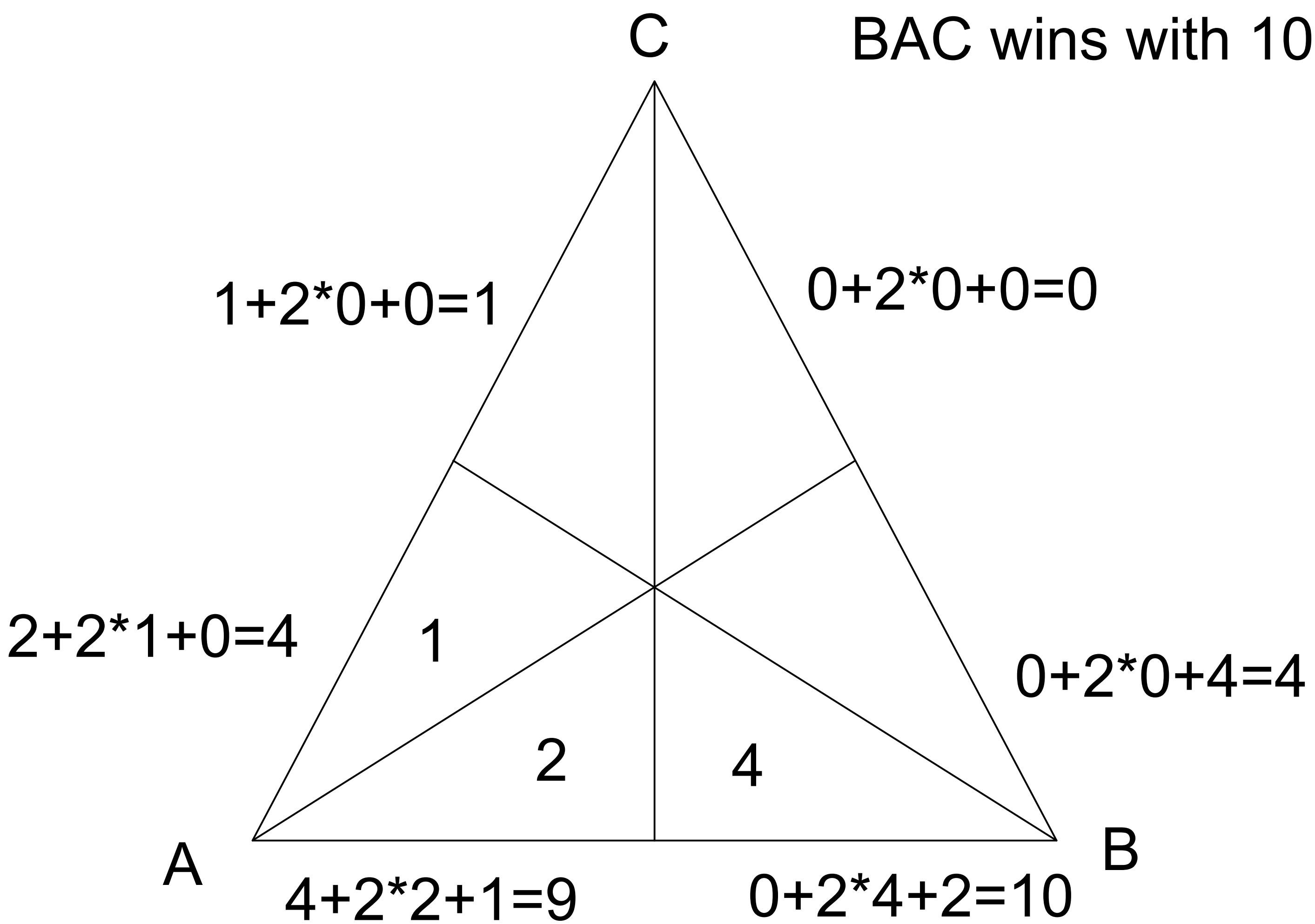
Things we’ll assume or ignore

- We’ll stick with three candidates in general, for the usual reasons.
- In the rules discussed, I won’t talk about how to modify claims for ties or no outcome situations.
- Indeed, our ‘social welfare functions’ may have outcomes that are a tie between several non-adjacent rankings!

Examples

The “1-2-1” rule:

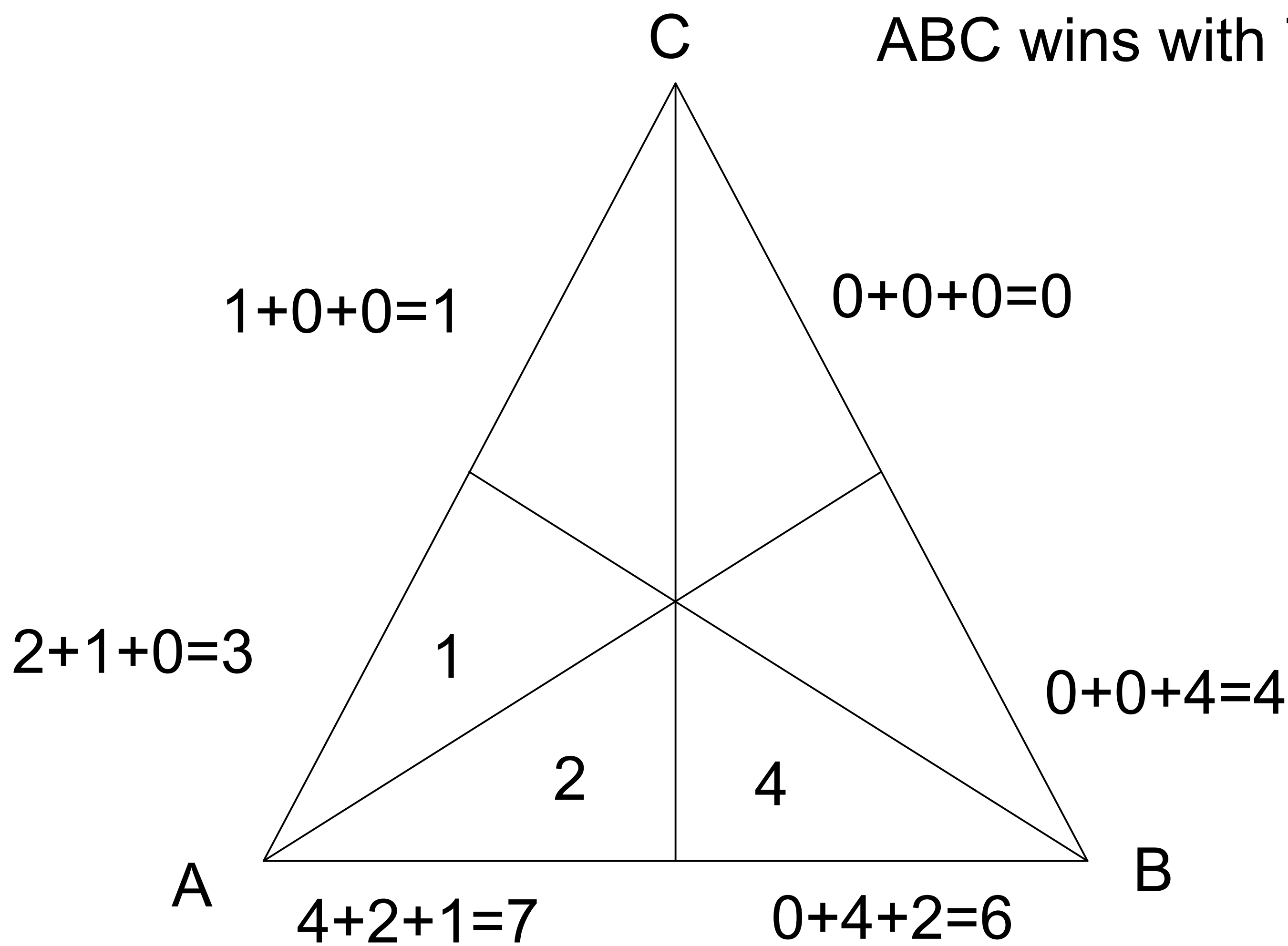
BAC wins with 10 points!



Examples

The “1-1-1” rule:

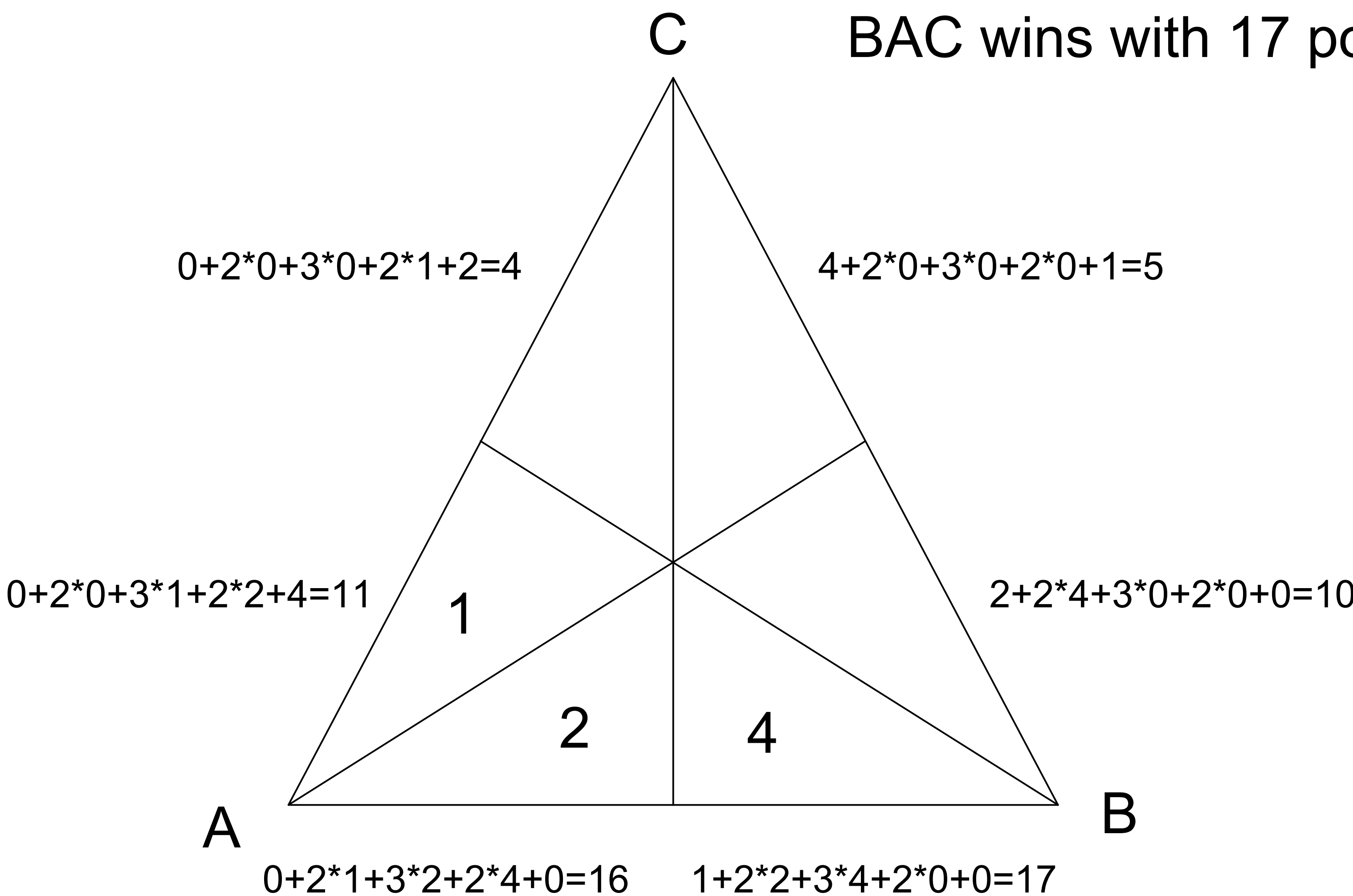
ABC wins with 7 points!



Examples

The “1-2-3-2-1” rule:

BAC wins with 17 points!

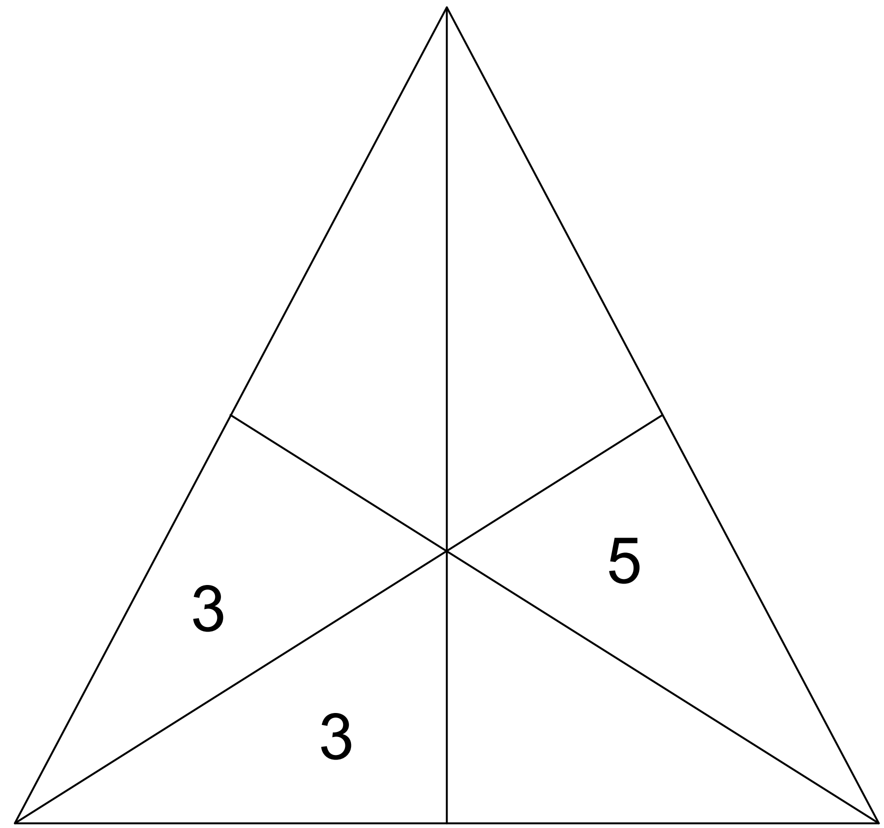


How different are these rules?

- We might expect such rules to disagree with various criteria or procedures:
  - No majority criterion
  - Disagree plenty with Borda Count
  - Not consistent *with respect to winners*
- Yet strangely consonant in other ways:
  - E.g., 1-2-1 tally for XYZ is Borda Count for X (winner) minus total having Z losing.

What about IIA?

- This profile has the Condorcet winner A (outcome A>B>C).
- The 1-2-1 system puts the Condorcet winner last (BCA receives 10 points).
- Rotate for loser first.



What about IIA?

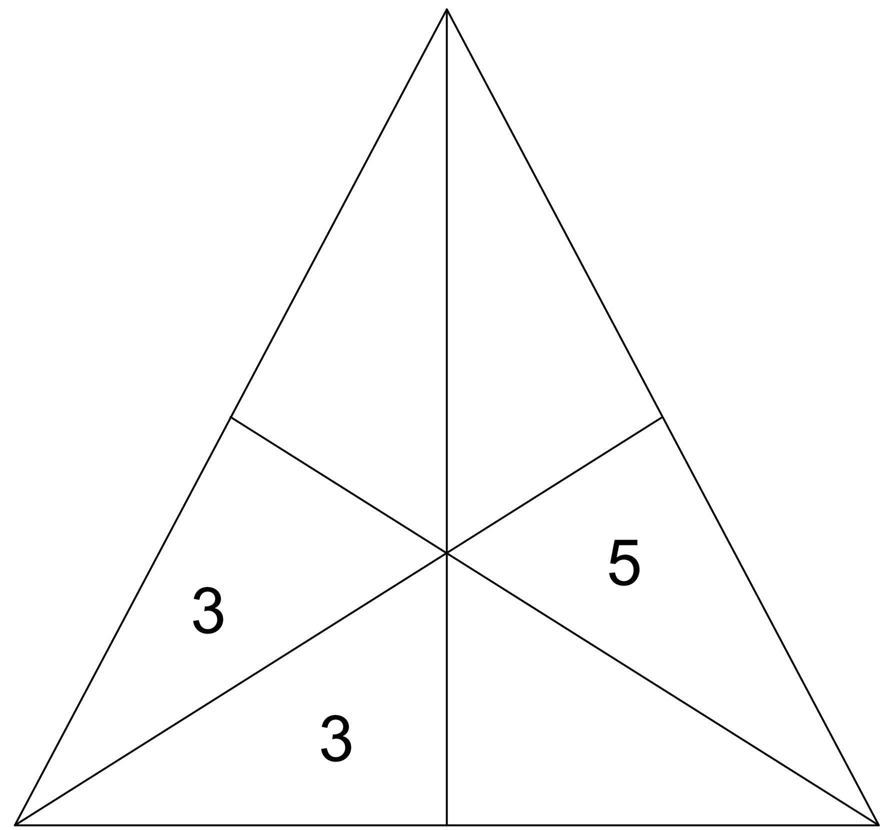
- One would hope that a rule connected to pairwise information would not violate Condorcet so much, so this seems bad.
- But this is not the only way to weakly respect IIA! There is a useful corollary to IIA (plus Pareto and anonymity):

  If a candidate is ranked first or last by all voters, the outcome ranks likewise.

What about IIA?

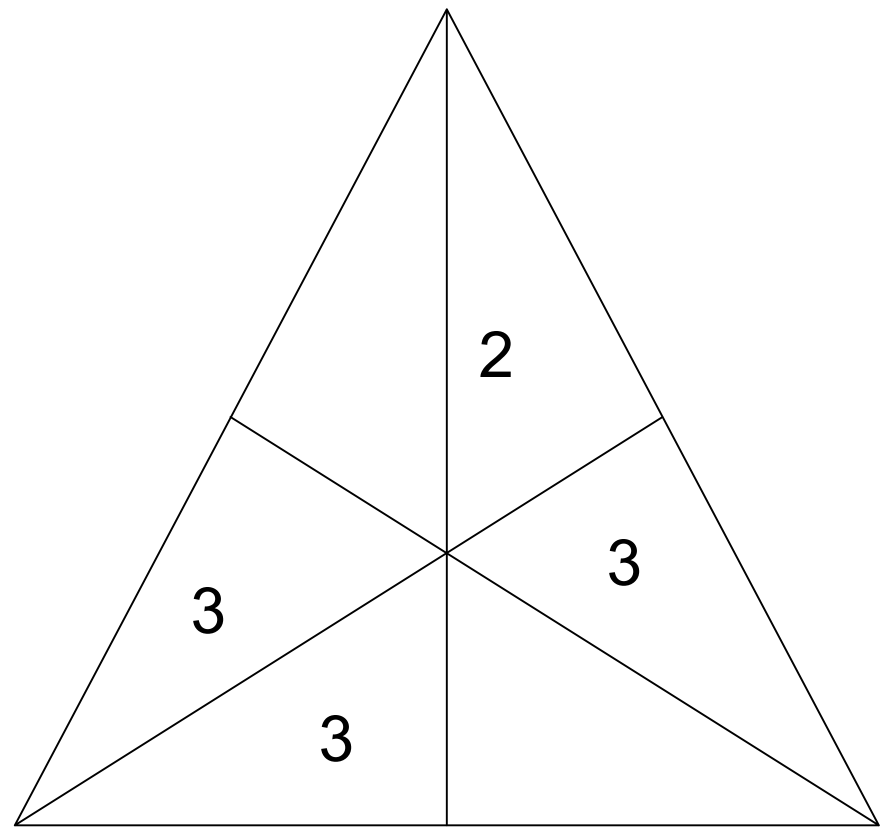
- Condorcet and 1-2-1 both obey this! They just sometimes have different winners.

- Note that the Borda Count yields BAC; it tries to average things more.



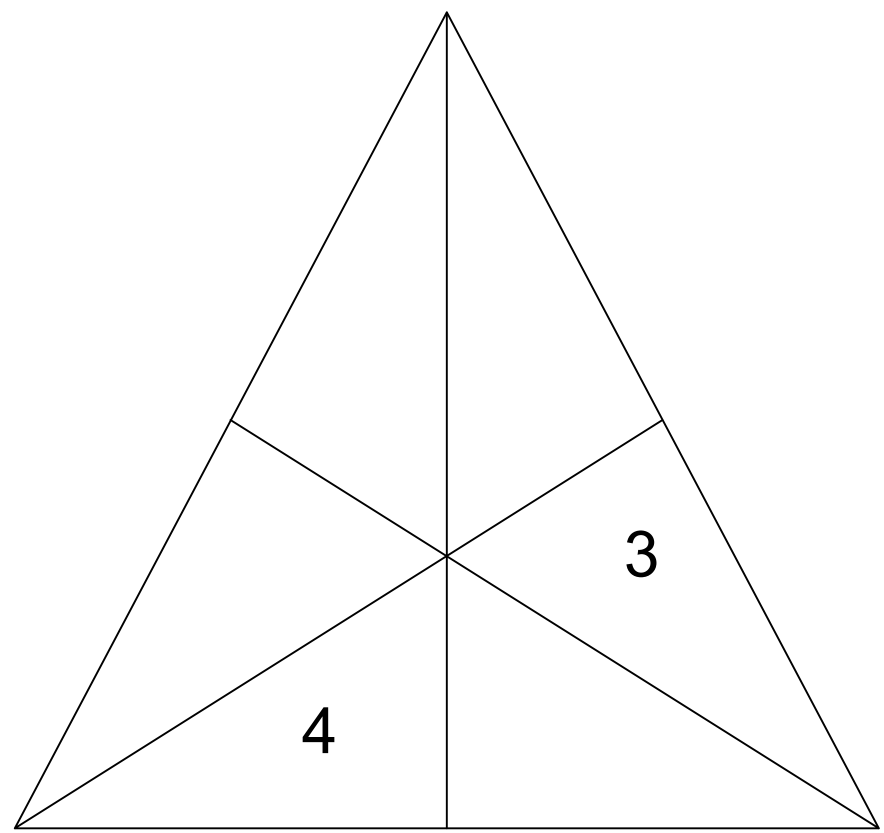
What about IIA?

- Switch to this profile:
- The Condorcet outcome is not changed, but now Borda and 1-2-1 both prefer A first.
- These methods value concentration of votes a lot.



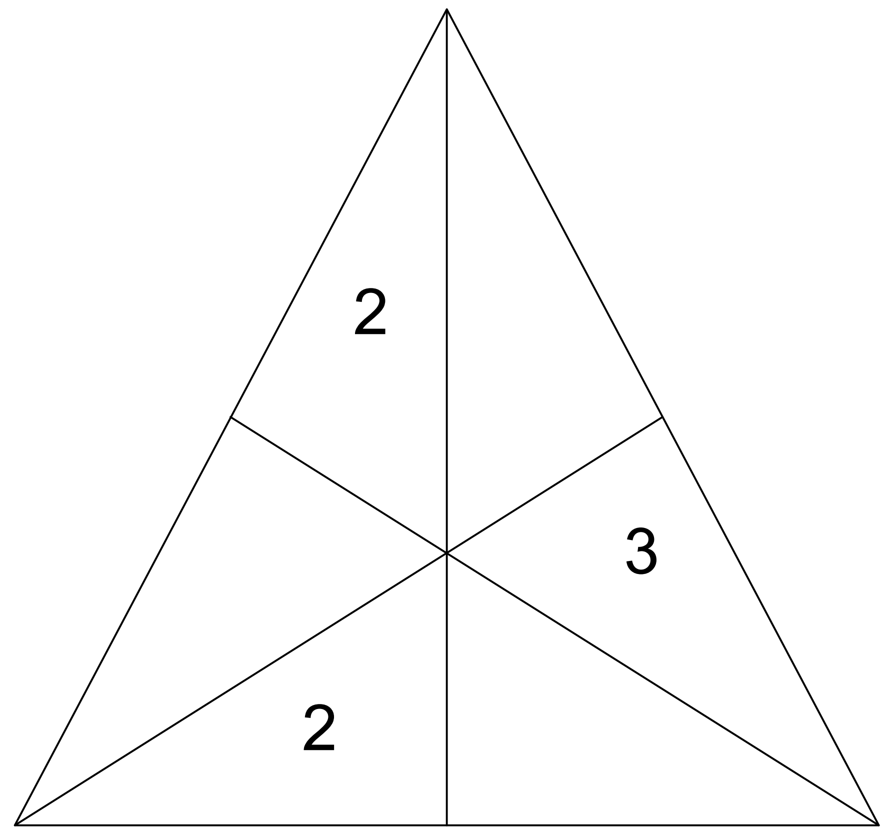
What about IBI?

- Using the 1-2-3-2-1 rule, this example yields ABC.
- Next we switch, but preserve A>B wishes *and* their intensity.



What about IBI?

- The outcome will change from ABC to BCA.
- But, the switch has *preserved* the “Condorcet type”, ABCABCA…

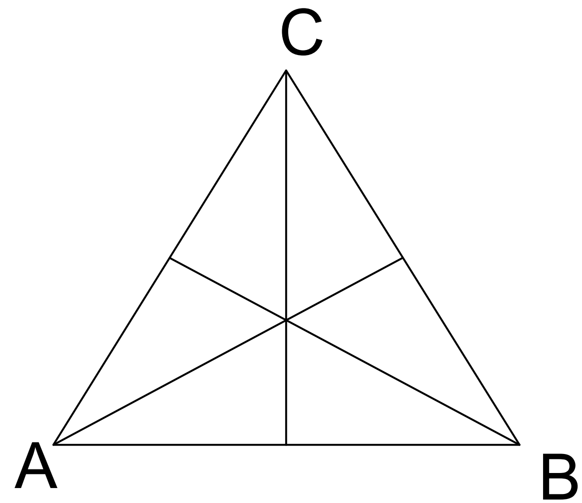


What about IBI?

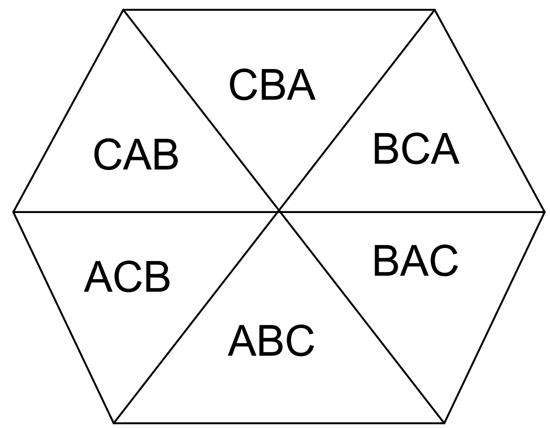
- Given a profile yielding XYZ with some procedures (e.g. 1-1-1 or 1-2-3-2-1), intensity X>Y preserving changes *either* keep X>Y *or* preserve the "Condorcet type" XYZXYZX…
- This bizarre (and *very* intriguing) behavior comes when we start removing hegemony of the winner from our procedures. We need a new source of intuition.

Symmetry for the Future

The representation triangle has only D_6 (or Σ_3) symmetry, while the hexagon has D_{12} symmetry.

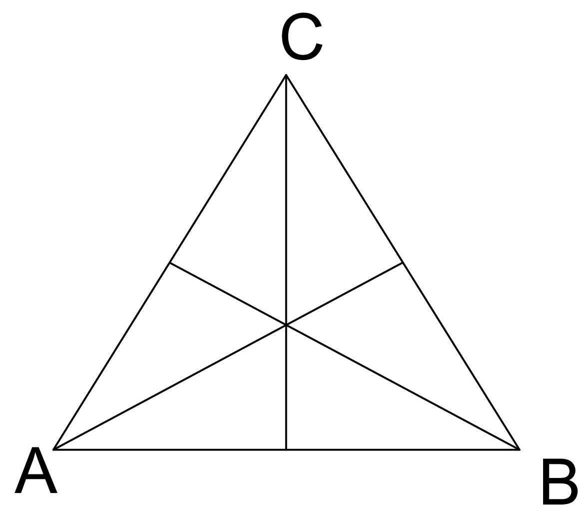


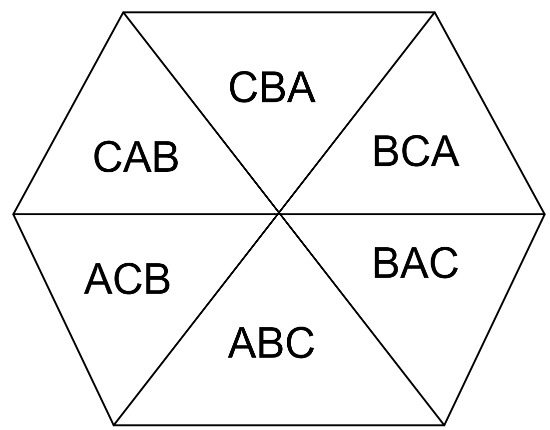
These procedures very naturally preserve this type of symmetry.



Symmetry for the Future

Hence, many symmetry-related methods of proof and discovery should be useful, whether combinatorial, geometric, or algebraic.





The recent methods of Orrison et al. in particular would seem to be very promising!