Due: Friday, February 14, 2020

Attached is a letter from Mr. Marton asking for my help. The problem he poses is so much like the mixing problems we examined in our class that I thought it would make a great project. The letter only includes some details, but it provides enough information to solve the problem he poses as long as we make some assumptions.

In the letter the concentrations are given in "ppt", or *parts-per-trillion*, the number of grams of pesticide per 1,000,000,000,000 grams of water. Knowing that 1,000 grams of water has a volume of 1 liter you can work out the initial quantity of pesticide in the pond and the amount entering the pond each day via the stream. You can assume that the pesticide concentration in the stream flowing into the pond is decaying exponentially, i.e., $c(t) = c_0 e^{-kt}$.

Assignment: I would like you to work in teams of two to solve this problem. *Each team will submit a single project report* containing the following three separate items:

1. A letter to Mr. Marton briefly describing the answer to his question. Please know that Mr. Marton is not a mathematician and only wants an answer to his question stated in a way he can understand. This should be formal business letter, written as if you were consultants contracted by Mr. Marton to address his problem.

2. A report that describes in detail your solution to the problem. The report should contain the following parts:

   - A cover sheet with the names of both partners.
   - A brief introduction to mixing problems and the differential equation that we use to model them.
   - A description of the model you used, including any assumptions you made. You should clearly state your initial value problem and describe the role of each term in the differential equation.
   - The work required to solve the initial value problem.
   - A graph of quantity of pesticide in the pond vs. time.
   - The work necessary to use your differential equation solution to answer Mr. Marton's question.
   - A conclusion or final discussion to wrap up your presentation.

   The report should read like a short paper and not merely be pages of mathematical work sandwiched between an introduction and a conclusion. It should flow naturally from one part to the next. Including the cover sheet and a page for the graph it will probably end up being between six and ten pages.

3. A description, signed by both partners, describing the contribution each of you made to the project. This should demonstrate that you both equally (but not necessarily identically) contributed to the project.

Dr. Jonathan Senning
Department of Mathematics and Computer Science
Gordon College
255 Grapevine Road
Wenham, MA 01984

Dear Prof. Senning,

I am writing you at the suggestion of a friend in my church in the hope that you can help me solve a problem. A former Gordon student, my friend says your differential equations class was the only 8am class that she looked forward to, and that after graduation she remembers many hours pleasantly spent working on differential equations modeling problems to relax at the end of a long day. Unfortunately the busyness of life in recent years has meant that she's not been able work many problems recently and so she thought I should go directly to you.

I have a small pond that is stocked with trout which I keep stocked with trout for fishing. Unfortunately in May of last year the stream which feeds the pond became contaminated with a pesticide as a result of some spraying on my neighbor's property. After some discussion (and my extending a standing invitation to come and fish), my neighbor agreed to not spray again so this should be a one-time problem. Right now, however, I have a contaminated water source and pond from which I've removed the trout stock. I would like to restock the pond as soon as the pesticide drops to a negligible level and I'm writing to ask for your helping figuring out when that will be.

The science faculty at our local high school helped me by measuring the pesticide level in the pond and the rate at which water is flowing in and out of the pond. The AP Calculus class took on the project of estimating the volume of the pond. They figured that the pond has a volume of 2,700,000 liters and suggested that we can ignore rain and evaporation since the pond level seems to remain fairly constant. The contaminated spring has a flow rate of 3,000 liters/day on average and there is another stream flowing out at the same rate. As I understand it, the pesticide is removed from the pond in two ways: decay (it has a half-life of 100 days) and by flowing out in the second stream. Fortunately the brand of pesticide used does not settle to the bottom but apparently remains dissolved in the water and should be fairly uniformly distributed.

The spraying occurred on May 16. On June 1st pesticide concentration in the pond was 1,850 ppt (parts-per-trillion). On the same date the stream flowing into the pond contained pesticide at a concentration of 20,000 ppt. Thirty days later on July 1st the pesticide concentration in the stream had dropped to 13,000 ppt. I didn't test the concentration in the pond itself on this second date; I hope that information won't be needed.

I'd like to know two things: “how bad did it get?” and “when can I restock?” I would therefore greatly appreciate any help you could give to find out:

1. what was the maximum pesticide concentration the pond and approximately when did it occur?
2. when will the concentration in the pond drop to 500 ppt?

I hope I've provided enough information, but please let me know if there is anything else you require.

Sincerely,
Taylor Marton
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