Number Theory: In Context and Interactive

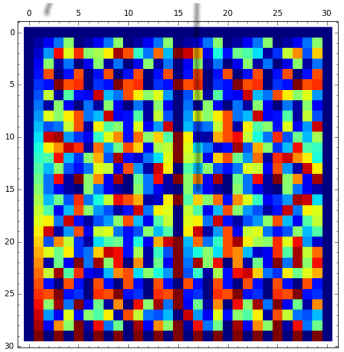
Karl-Dieter Crisman

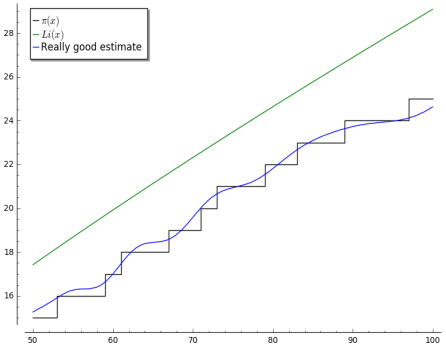
Gordon College

Number Theory CPS
Joint Mathematics Meetings: January 12 2018, San Diego, CA

Overview

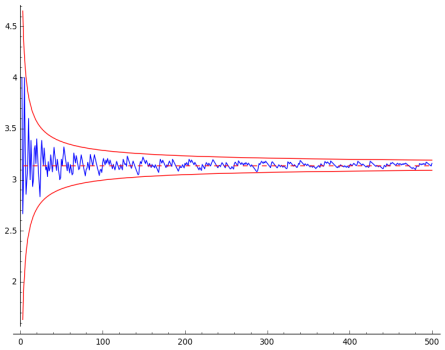
Number Theory

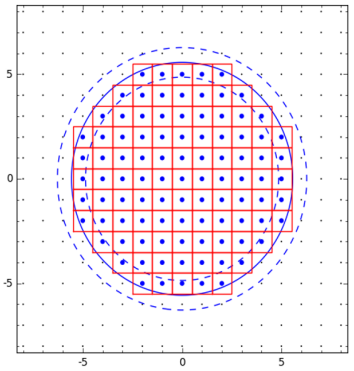




What do you want your number theory course to be?

In Context and Interactive





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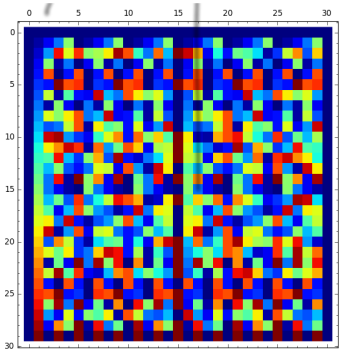

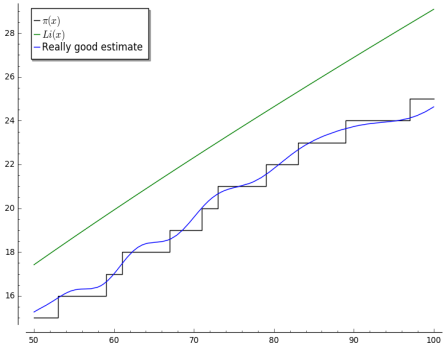

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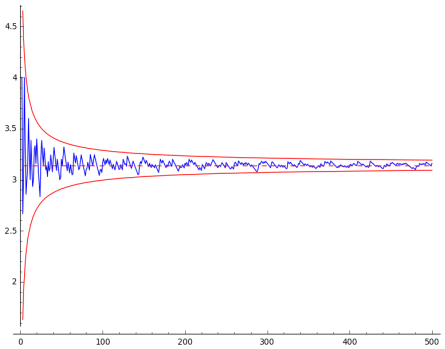

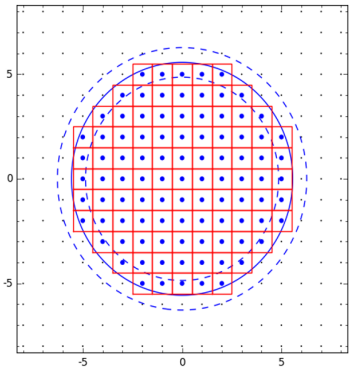

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My answer:

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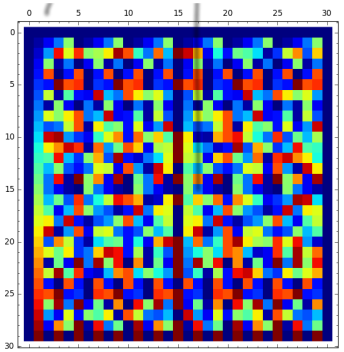

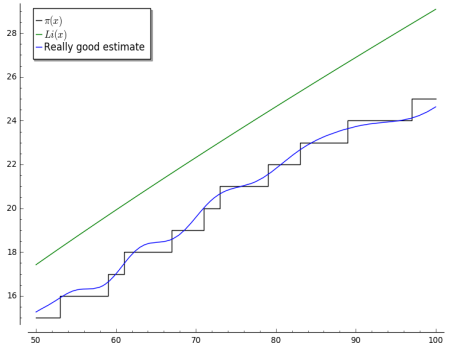

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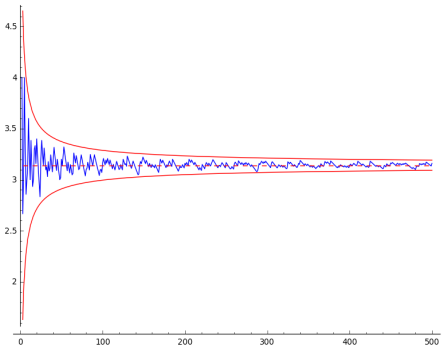

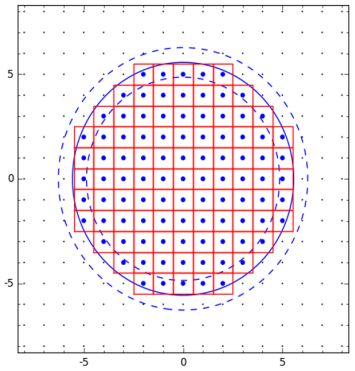

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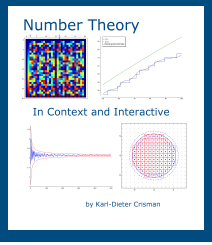
My answer:

- In context
- Interactive

`http://math.gordon.edu/ntic/`



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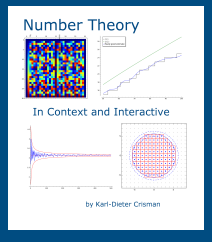
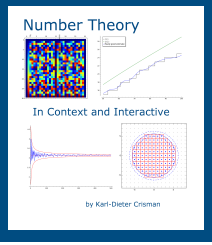


Overview

My context

- Around 1500 students, about 3% math majors
- Number Theory as 300-level elective (4/8, not req. for ed)
- Four-credit course, can assume intro to proof and calculus
- No more than 15 students



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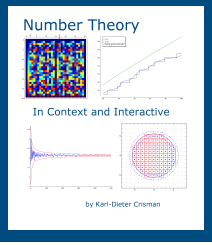

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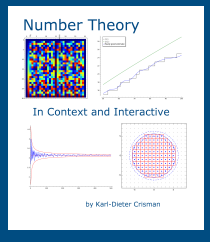
My goals

- See the *unity* of mathematics
- Have opportunity to *explore*
- Harness/be aware of modern computational power



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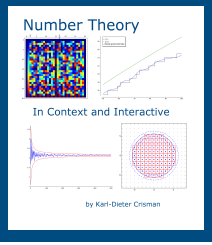
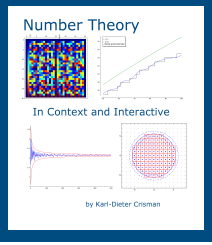
These are only *my* goals.



Overview

These are only *my* goals.

- Unity of mathematics – *In Context*
- Exploring with modern computational power – *Interactive*



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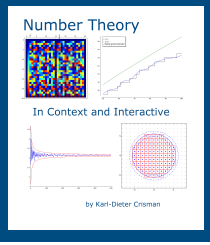

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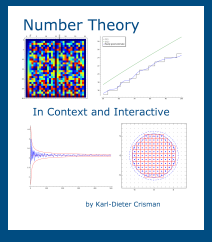
Other possible goals somewhat compatible (I do some of each):

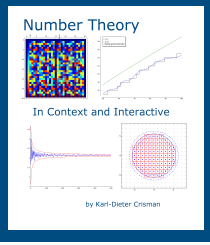
- Proof practice/development
- True IBL
- Introduction to programming
- Introduction to cryptography/cryptology



Overview

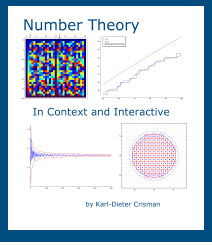
A few more boring details:

- Computation provided by SageMath; much use of its `@interact` function with number-theoretic functionality
- Authoring began as worksheets, but eventually ported to PreTeXt (formerly known as Mathbook XML)
- Students required to buy print to *write* in, use online for reference – both were heavily used, though more heavily by different people



In Context and Interactive

What does *In Context* mean?



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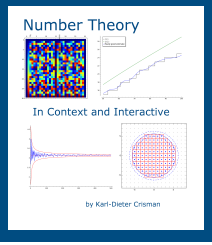
What does *In Context* mean?

- Explicitly making connections to previous coursework in standard material
- Choose as many topics with connections to previous coursework as possible



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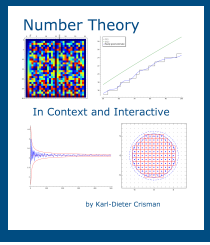

In Context and Interactive

What does *In Context* mean?

- Explicitly making connections to previous coursework in standard material
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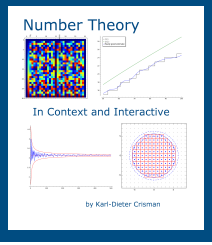
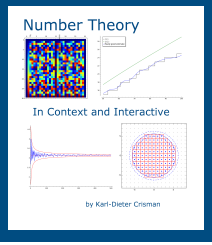
What does *Interactive* mean?

- Explicitly asking students to use computation to extend their previous abilities and examine large cases for patterns
- Choose as many topics with the opportunity for exploration *and/or understanding* with technology as possible



In Context

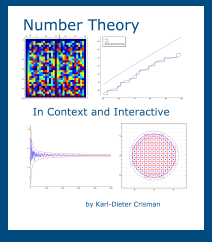
What sort of explicit connections?



In Context

What sort of explicit connections?
Many are ones which should be familiar and likely to show up.

- Abstract Algebra – Defining groups to make work with Euler's theorem, primitive roots easier
- Geometry – Solutions of linear Diophantine equations as lines intersecting the integer lattice
- Analysis – Gauss estimating primes as $\int_2^n \frac{dt}{\log(t)}$



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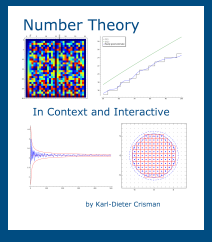

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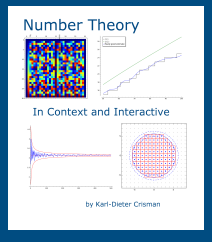
- Algebra – Group operation of adding rational points on a *conic section*
- Geometry – Proving every prime of the form $4k + 1$ can be written as a sum of squares 'with Minkowski'
- Analysis – *Proving* limits involving arithmetic functions like number of ways to write as a sum of squares



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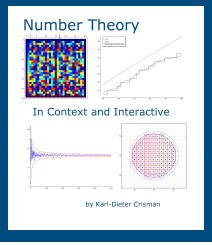
Calculus has many more connections than we might recall!



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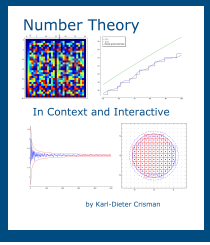
Calculus has many more connections than we might recall!

- Tangent lines to curves help find new rational points
- "Newton's method" helps find solutions modulo p^2 from solutions modulo p
- Combining integrals and limits to find estimates of the 'long-term average' of the number of/sum of divisors functions



In Context

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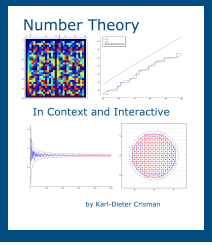
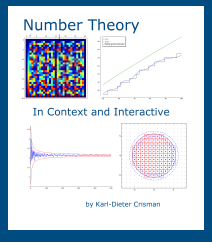


In Context

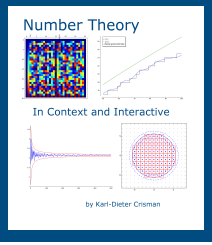
There are other connections, and combinations.

- Calculus and geometry combine in proving the 'average value' of τ
- Combinatorics really provides most beautiful proof of Fermat's Little Theorem
- Linear Algebra and systems of linear *congruences* can be expanded on quite a bit



Interactive

Much exploration can still be done by hand.



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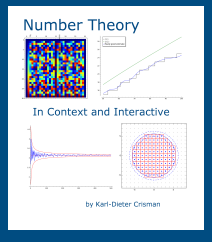
"Question 9.2.2. Do you see any patterns in the value of $\phi(n)$?"



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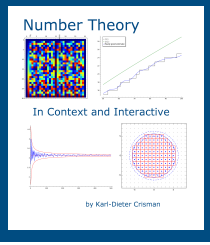

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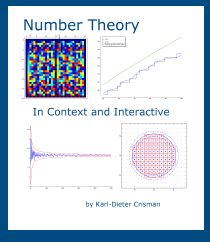

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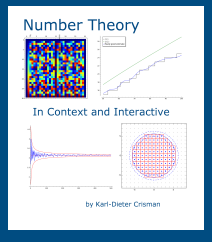
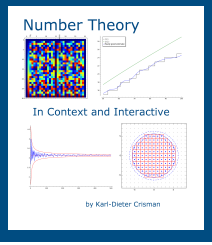
Just as importantly, interactive versions of geometric proofs can quickly elucidate what the *important* visual features are where static versions are far harder to parse.



Demos

Here there be warnings against trying demos without internet.



Concluding Unscientific Postscript

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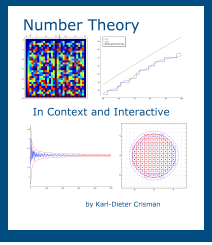

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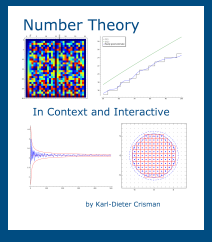
This is an advertisement for number theory that:

- Incorporates bits of algebra, calculus, geometry, and more ...
- Has lots of exploration and interactivity

Regardless of platform.



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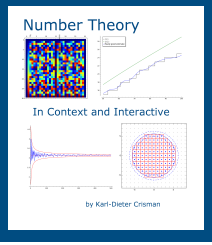

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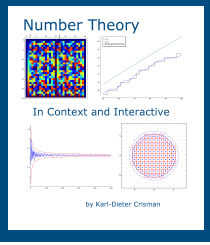
But of course it's also an advertisement for:

- The authoring tool, PreTeXt
- SageMath mathematics computation software
- Open licenses for your texts



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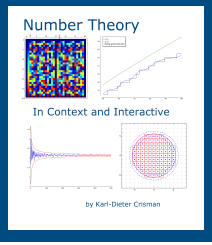

Thanks!

Thanks are due to:

- You for coming!
- The organizers for accepting my abstract
- The SageMath and PreTeXt communities for a lot of support and opportunities/funding to learn more about each of them
- Many many authors of far better number theory books whose ideas I have strived my best to interpret in new ways for my unique classes
- Most importantly, the dozens of students I have tried this on in MAT 338 Number Theory since 2007.